FRAME OF REFERENCE

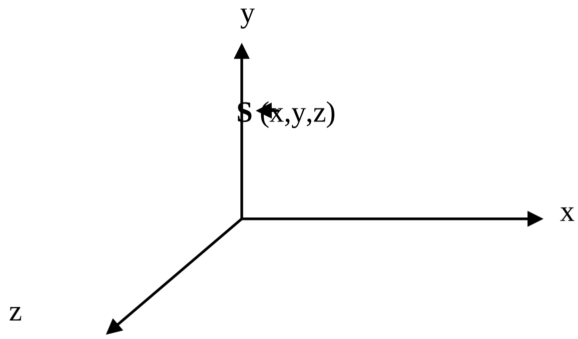
A system related to which measurement are done, is called frame of reference.

Or

A system of co-ordinate axes which defines the position of a particle ***or*** an event in two ***or*** three dimensional space is called frame of reference.

A co-ordinate axes (x,y,z) are considered as a frame of reference.

Where 'S' is frame of reference.



The essential requirement of a frame of reference is that it should be rigid.

A reference frame, with four co-ordinates xyz and t is referred to as a space-time frame and give the complete identification of an event in frame of reference.

INERTIAL FRAME OF REFERENCE

A frame of reference where law of inertia are valid (Newton's I^{st} and II^{nd} law hold good), is called as inertial frame of reference or Newtonian or Galilean frame of reference.

An inertial frame may be at rest or moving with constant velocity. So I^{st} law defines and II^{nd} law identifies that it is non-accelerated frame of reference.

This means ,

$$\mathbf{a} = d^2\mathbf{r}/dt^2 = 0 \qquad \text{because } \mathbf{F} = m\mathbf{a} = 0$$

Or

$$d^2x/dt^2 = d^2y/dt^2 = d^2z/dt^2 = 0$$

Example : **-** A frame attached to the Earth is approximately treated as inertial frame of reference . But strictly speaking that Earth is not inertial frame of reference.

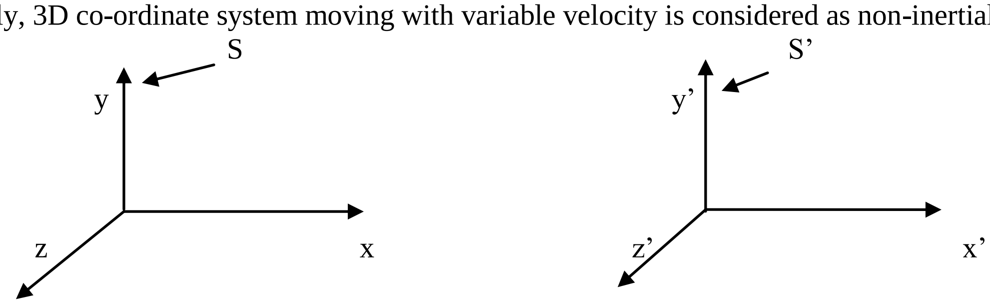
NON INERTIAL FRAME OF REFERENCE

A frame of reference where law of inertia (Newton's I^{st} and II^{nd} law) are not valid , is called non inertial frame of reference.

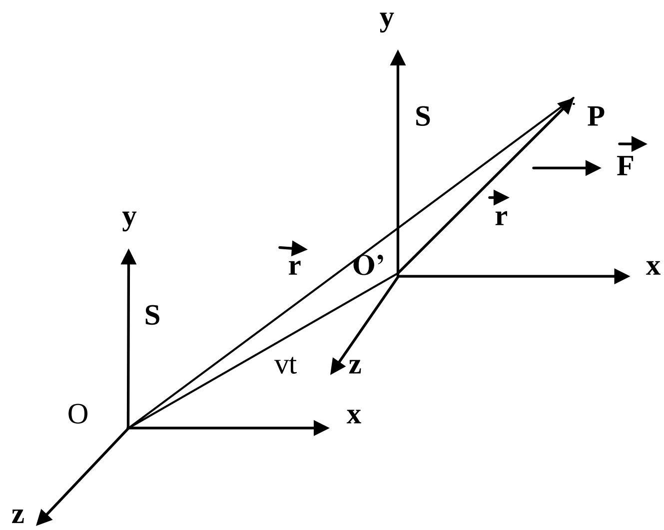
or

A frame of reference moving with variable velocity or constant acceleration is called as non- inertial frame of reference.

Generally, 3D co-ordinate system moving with variable velocity is considered as non-inertial frame of reference.



'S' is non-inertial frame. Consider two frames of reference S and S. S is inertial where as S is non-inertial frame of reference.



In Non-inertial frame S' a fictitious force starts acting.

Consider a particle of mass moving in the space, when measured from S, let 'a' be its acceleration.

Force acting on the particle from S,

$$\mathbf{F} = m\,\mathbf{a_i} \qquad \text{(i)}$$

Force acting on the particle from S',

$$\mathbf{F'} = m\,\mathbf{a_i} + \text{imaginary force (fictitious force)} \qquad \text{(ii)}$$

From (i) and (ii), we observed that

$$\mathbf{F} \neq \mathbf{F'}$$

As **F** and **F'** are not equal, so Newton's Ist law of motion is not valid.

Example : - All accelerated frame of reference is an example of non-inertial frame of reference.

Any frame attached to the Earth is another example of non-inertial frame of reference.

Note:- The force experienced by the particle due to acceleration of non-inertial frame of reference is called Fictitious force.

NEWTON'S LAW OF MOTION

Newton gave three laws of motion which are known as ***law of mechanics***.

Ist law: - A body must continue in its state of rest or uniform motion, unless not disturbed by some external influence.

This is also known as ***'law of inertia'***.

Every body has the property of inertia. This inertia is different for different bodies.

IInd law: - The time-rate of change of momentum is proportional to the impressed force,

$$\boldsymbol{F} = d\mathbf{p}/dt = d(m\mathbf{v})/dt$$

$$\mathbf{F} = m\mathbf{a} \text{ where } d\mathbf{v}/dt = \mathbf{a}$$

Force = Mass . Acceleration

IIIrd law: - 'For every action there is an equal and opposite reaction'.

But action and reaction forces act on different bodies.

Let 1 and 2 two bodies are interacting mutually, then

$$\mathbf{F_{12} = - F_{21}}$$

i.e. Force on Ist body due to IInd = **-** Force on IInd body due to Ist

But we know that

$$\boldsymbol{F} = m\mathbf{a} = m\, d(\mathbf{v})/dt \qquad \text{(II}^{nd}\text{ law)}$$

$$\mathbf{F_{12}} = m_1\, d\mathbf{v_1}/dt \text{ and } \qquad \mathbf{F_{21}} = m_2\, d\mathbf{v_2}/dt$$

So, according to IIIrd law

$$m_1 d\mathbf{v_1}/dt = - m_2\, d\mathbf{v_2}/dt$$

if $\mathbf{a_1} = |d\mathbf{v_1}/dt|$ and $\mathbf{a_2} = |d\mathbf{v_2}/dt|$ denote the magnitude of acceleration,
then $m_1\, \mathbf{a_1} = m_2\, \mathbf{a_2}$ or $m_2 = m_1\, \mathbf{a_1} / \mathbf{a_2}$.

Thus Newton's IIIrd law defines mass unequally.

RECTILINEAR MOTION

If a particle changes its position with time along straight line, then its motion is said to be rectilinear motion.

Let the particle has velocity **u** at the origin (x=0) at time (t=0) and moving with constant acceleration **a** then the final velocity v at time t is given,

From definition of acceleration,

$$\mathbf{a} = d\mathbf{v}/dt$$

$$d\mathbf{v} = \mathbf{a}\, dt$$

$$\int_u^v d\mathbf{v} = \int_0^t \mathbf{a}\, dt$$

$$\mathbf{v} - \mathbf{u} = \mathbf{a}t \qquad \text{(i)}$$

Equation (i) can be written as

$$\mathbf{v} = dx/dt = \mathbf{u} + \mathbf{a}t$$

$$dx = (\mathbf{u} + \mathbf{a}t)\, dt$$

$$\int_0^s dx = \int_0^t (\mathbf{u} + \mathbf{a}t) dt$$

$$s = \mathbf{u}t + \tfrac{1}{2}\,\mathbf{a}t^2 \qquad \text{(ii)}$$

From equation (i) $\mathbf{v}^2 = (\mathbf{u} + \mathbf{a}t)^2$

$$\mathbf{v}^2 = \mathbf{u}^2 + \mathbf{a}^2t^2 + 2\mathbf{ua}t$$

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\mathbf{a}(\mathbf{u}t + \tfrac{1}{2}\,\mathbf{a}t^2)$$

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\mathbf{a}s \qquad \text{(iii)}$$

Equation (i), (ii) and (iii) are called equation of rectilinear motion with constant acceleration.

WEIGHTLESSNESS

It is the state when apparent weight of the body becomes zero.

It may arise when goes downward due to gravity.

So equation of motion of lift,

$$\textbf{\textit{Net Force}} = \textbf{\textit{Mass . Acceleration}}$$

$$m\mathbf{g} - R = m\mathbf{a} \qquad \text{(i)}$$

But $\mathbf{a} = \mathbf{g}$, $\quad m\mathbf{g} - R = m\mathbf{g}$

$$R = 0$$

So apparent weight of body will be zero. Under this situation bodies of different weight have same apparent weight = 0 and all bodies float.

FRICTIONAL FORCE

When the surface of body slides over the surface of another body, each body exerts a force on each body opposite to the direction of its motion relative to the other. It is denoted by f_s.

LAW F LIMITING FRICTION

The limiting (maximum) static frictional force depends upon the nature of the surfaces in contact. It does not depend upon the size or area of the surfaces. For the given surfaces, the limiting friction force f_s is directly proportional to the normal reaction R.

$$f_s \;\alpha\; R$$

$$f_s = \mu_s R$$

Where the constant of proportionality 'μ_s' is called the ***'Coefficient of static friction'***.

Dynamic (Kinetic) friction

The force acting between the surface in relative motion is called the 'dynamic frictional force' (f_k) , which is less than the limiting force of static friction f_s .

When the block is in uniform motion the force of dynamic friction is

$$f_k = \mu_k R$$

$$\mu_k < \mu_s$$

ANGLE OF FRICTION

In the state of limiting friction , the angle in which the resultant of the limiting frictional force f_s and the normal reaction R makes with the normal reaction R is called the 'Angle of friction'. If the angle is θ_s ,

$$\tan \theta_s = f_s / R$$

But $\quad f_s / R = \mu_s$

$$\tan \theta_s = \mu_s$$

MOTION ON SMOOTH PLANE

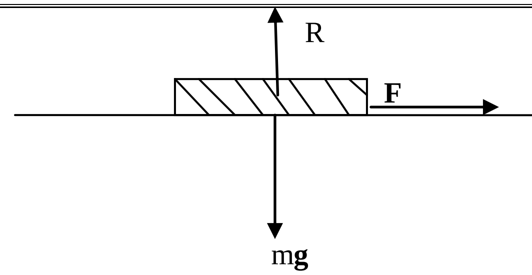
Let a body of mass 'm' is placed on a smooth surface. The first force (weight) which acts vertically downward. The second force is the reaction force 'R' which always acts normally to the body.

Now let a force '**F**' is applied horizontally

so that the body moves with acceleration '**a**'

along the direction of force.

Then



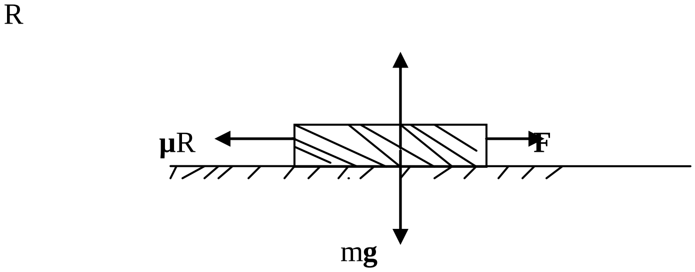
Eq. of motion

(i) $R = m\mathbf{g}$

(ii) $\mathbf{F} = m\mathbf{a}$

MOTION ON A ROUGH PLANE

In this case a frictional force μR also acts which opposes motion. Hence it is directed opposite to the direction of motion. Therefore



Eq. of motion of body

(i) $R = m\mathbf{g}$

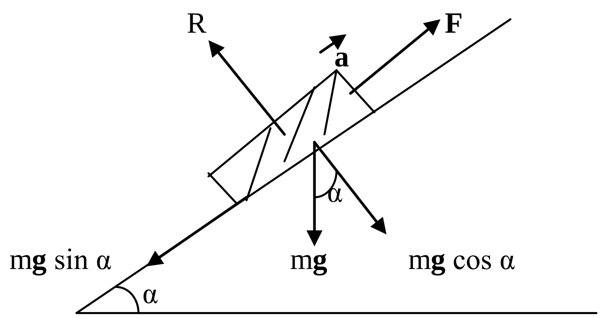
(ii) Net force , $\mathbf{F} - \mu R = m\mathbf{a}$

$\mathbf{F} - \mu m\mathbf{g} = m\mathbf{a}$

MOTION ON A SMOOTH INCLINED PLANE

Let a body of mass 'm' is placed on an inclined plane having angle of inclination α.

The first force (weight) acting vertically downward. The second force will be reaction force 'R' acting normally to the body. Now resolve the 'm**g**' into two components 'm**g** cos α' & 'm**g** sin α'.



(a) **Upward motion-** Now let a force '**F**' is Applied on the body so that it moves up with acceleration '**a**'.
Then eq. of motion of body

(i) $R = m\mathbf{g} \cos \alpha$

(ii) Net force, $\mathbf{F} - m\mathbf{g} \sin \alpha = m\mathbf{a}$

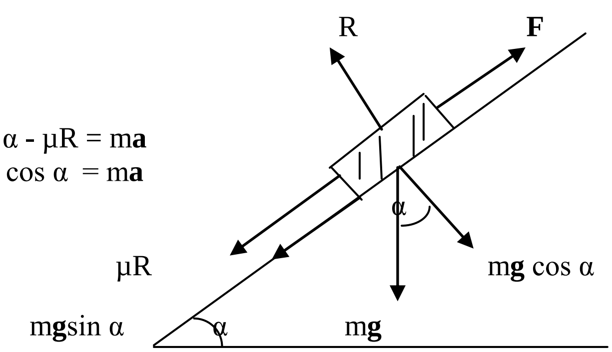
(b) **Downward motion-**
Equation of motion of body

(i) $R = m\mathbf{g} \cos \alpha$

(ii) Net force, $\mathbf{F} + m\mathbf{g} \sin \alpha = m\mathbf{a}$

MOTION ON A ROUGH INCLINED PLANE

In this case of rough inclined plane, an additional frictional force acts opposite to the direction of motion



Upward motion-

Eq. of motion of body

(i) $R = m\mathbf{g} \cos \alpha$

(ii) Net force, $\mathbf{F} - m\mathbf{g} \sin \alpha - \mu R = m\mathbf{a}$

$\mathbf{F} - m\mathbf{g} \sin \alpha - \mu m\mathbf{g} \cos \alpha = m\mathbf{a}$

Downward motion-

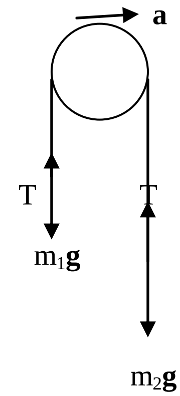
Eq. of motion of body

(i) $R = m\mathbf{g} \cos \alpha$

(ii) Net force, $\mathbf{F} + m\mathbf{g} \sin \alpha - \mu R = m\mathbf{a}$

$\mathbf{F} + m\mathbf{g} \sin \alpha - \mu m\mathbf{g} \cos \alpha = m\mathbf{a}.$

MOTION ATTACHED TO TWO MASSES



Let two masses m_1 & m_2 ($\mathbf{m_1 < m_2}$) are Attached with mass less, inextensible String which passes over a weightless & frictionless pulley. Let m_2 moves downward with acceleration '**a**'.

Hence m_1 will move upwards with same acceleration.

<u>Eq. of motion m_1</u>, $T - m_1\mathbf{g} = m_1\mathbf{a}$ (i)

<u>Eq. of motion m_2</u>, $m_2\mathbf{g} - T = m_2\mathbf{a}$ (ii)

From eq. (i) + (ii), we have

$\mathbf{a} = (m_2 - m_1 / m_2 + m_1)\mathbf{g}$ (iii)

Putting the value of 'a' in eq. (i) or (ii), we have

$T = 2\,(m_1 m_2 / m_1 + m_2)\mathbf{g}$

So if m_1 & m_2 are known we can get 'a' and T.

MOTION ATTACHED MASSES ON A DOUBLE INCLINED PLANE

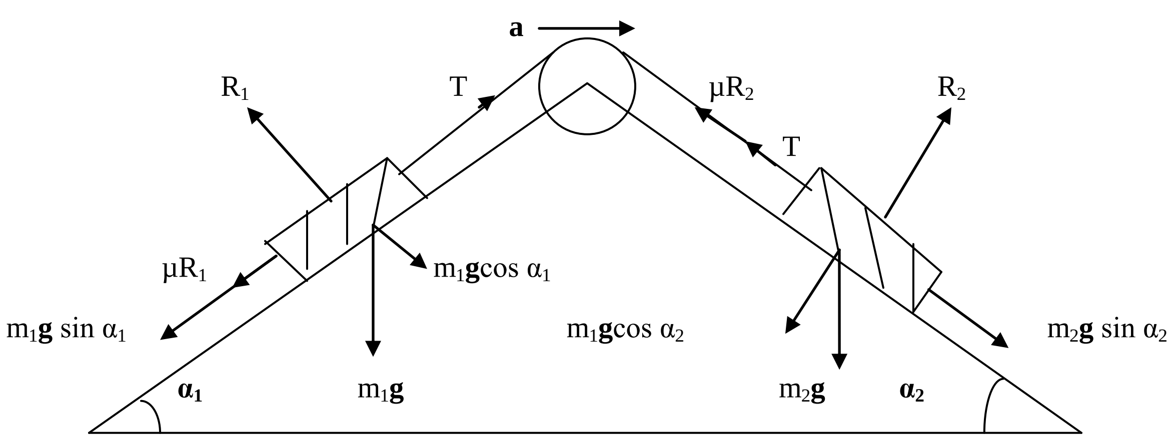
Let two mass m_1 & m_2 ($\mathbf{m_1 < m_2}$) are attached with a mass less, inextensible string which passes over a weightless & frictionless pulley.

Let this pulley is placed on the top of a double inclined plane has coefficient of friction 'μ'. Now if a force '**F**' is applied on mass m_2 such that it moves downward with acceleration '**a**'.

<u>Eq. of motion m_2</u> (i) $R_2 = m_2\mathbf{g}\cos\alpha_2$

(ii) Net Force, $\mathbf{F} + m_2\mathbf{g}\sin\alpha_2 - \mu R_2 - T = m_2\mathbf{a}$

$\mathbf{F} + m_2\mathbf{g}\sin\alpha_2 - \mu\, m_2\mathbf{g}\cos\alpha_2 - T = m_2\mathbf{a}$



<u>Eq. of motion m_1</u> (i) $R_1 = m_1\mathbf{g}\cos\alpha_1$

(ii) Net Force, $T - m_1\mathbf{g} \sin \alpha_1 - \mu R_1 = m_1\mathbf{a}$

$T - m_1\mathbf{g} \sin \alpha_1 - \mu\, m_1\mathbf{g} \cos\alpha_1 = m_1\mathbf{a}.$

LABORARTORY AND CENTRE OF MASS FRAMES OF REFERENCE

In the explanation of collision between two particles we shall come across the laboratory system or laboratory frame of reference and center of mass system of center of mass of reference.

A reference frame is the space determined by a rigid body regarded as the base. The rigid body is supposed to extend in all directions as far as necessary. A point in space is located by the three coordinates taken with respect to the origin of the reference system.

If the origin of the reference system is a point rigidly fixed to the laboratory it is known as the laboratory frame.

The laboratory frame is inertial so long as earth is taken to be an inertial frame.

Centre of mass system (Frame of reference): If the origin of the reference system is a point rigidly fixed to the center of mass of a system of particles on which no external force is acting it is known as the center of frame of reference.

In the center of mass frame of reference the position vector of the center of mass R = 0 as the center of mass is itself the origin of the reference system.

Therefore velocity of center of mass $v = dr/dt = 0$

And linear momentum $P = m\,v$ of the system is also = 0. Hence it is known as a zero momentum frame.

Collision

It is defined as the phenomenon of the change in the velocities of bodies during the very small fine interval of their contact.

Or

If two bodies come together and interact with strong for a short time, then the event is called as collision. The collision is termed ***scattering*** if the nature of particles does not change after collision.

In collision process it is not at all assumed that the particles actually come in contact but if two particles may not even touch each other, they must be said to collide. e.g., in the case of one alpha particle and a nucleus of Gold.

In collision Linear Momentum is conserved.

$\mathbf{P}_{(before)} = \mathbf{P}_{(after)}$ **or** $\Rightarrow \mathbf{P} = \text{constant}$

Kinds of collision

Elastic collision- A collision is said to be an elastic if

i) the final particles after collision are the same as the initial particles before collision
ii) the sum of the K.E. and linear momentum of the colliding particles after collision is same as before collision.

e.g. -Collision between electrons, protons, alpha particles.

-Collision between atoms and nuclei.

-Collision between gas molecules.

<u>Inelastic collision-</u> A collision is said to be inelastic if

i) the final particles after collision are the same as the initial particles before collision
ii) the sum of the K.E of the colliding particles after collision is either more or less than the sum of K.E of the particles before collision.

K.E. is not conserved but total energy and momentum are still conserved.

e.g. – A bullet remains embedded in a target.

Kinds of Inelastic collision

In this case K.E may be decreased or increased.

a. **Endoergic Collision:** For bodies of macroscopic size the loss of kinetic energy occurs as heat, sound etc but in the case of atoms, molecules etc the atoms may absorb a part of K. E and move into an excited state. The K. E of the particles is then reduced.

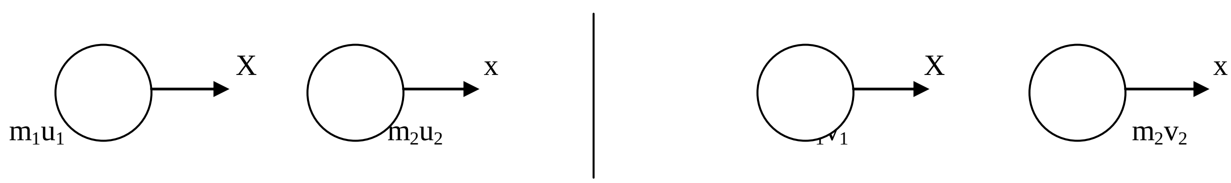
$$\tfrac{1}{2} m_1\mathbf{u_1}^2 + \tfrac{1}{2} m_2\mathbf{u_2}^2 = \tfrac{1}{2} m_1\mathbf{v_1}^2 + \tfrac{1}{2} m_2\mathbf{v_2}^2 + E$$

where E is excitation energy. A collision in which the K. E of the final particles is less than the K. E of the initial particles is known as endoergic collision.

b. **Exoergic Collision:** A collision in which the K. E of the final particles is more than the K. E of the initial particles is known as exoergic collision.

<u>Elastic collision in one dimension</u>

Let us consider two particles A and B of masses m_1 & m_2 moving with velocities u_1 and u_2 collide head on and after collision their velocities becomes v_1 and v_2.



Before collision After collision

Then according to conservation law of momentum,

$$m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$$

$$-(m_1\mathbf{v_1} - m_1\mathbf{u_1}) = (m_2\mathbf{v_2} - m_2\mathbf{u_2})$$

$$- m_1(\mathbf{v_1} - \mathbf{u_1}) = m_2(\mathbf{v_2} - \mathbf{u_2}) \qquad \text{(i)}$$

And according to conservation of K.E.,

$$\tfrac{1}{2} m_1\mathbf{u_1}^2 + \tfrac{1}{2} m_2\mathbf{u_2}^2 = \tfrac{1}{2} m_1\mathbf{v_1}^2 + \tfrac{1}{2} m_2\mathbf{v_2}^2$$

$$m_1\mathbf{u_1}^2 + m_2\mathbf{u_2}^2 = m_1\mathbf{v_1}^2 + m_2\mathbf{v_2}^2$$

$$- m_1(\mathbf{v_1}^2 - \mathbf{u_1}^2) = m_2(\mathbf{v_2}^2 - \mathbf{u_2}^2)$$

$$- m_1(\mathbf{v_1} + \mathbf{u_1})(\mathbf{v_1} - \mathbf{u_1}) = m_2(\mathbf{v_2} + \mathbf{u_2})(\mathbf{v_2} - \mathbf{u_2}) \quad \text{(ii)}$$

Dividing (ii) by (i), we get $(\mathbf{v_1} + \mathbf{u_1}) = (\mathbf{v_2} + \mathbf{u_2})$

$$(\mathbf{u_1} - \mathbf{u_2}) = -(\mathbf{v_1} - \mathbf{v_2}) \quad \text{(iii)}$$

Therefore in a perfectly elastic collision the relative velocity remains unchanged in magnitude, but it is reversed in direction.

Velocity after collision:- From eq. (iii) , we get

$$\mathbf{v_1} = \mathbf{v_2} + \mathbf{u_2} - \mathbf{u_1} \quad \text{(iv)}$$

$$\mathbf{v_2} = \mathbf{v_1} + \mathbf{u_1} - \mathbf{u_2} \quad \text{(v)}$$

So from eq. (i), we get

$$- m_1(\mathbf{v_1} - \mathbf{u_1}) = m_2(\mathbf{v_1} + \mathbf{u_1} - \mathbf{v_2} - \mathbf{u_2})$$

$$\mathbf{v_1} = (m_1 - m_2 / m_1 + m_2)\,\mathbf{u_1} + (2\, m_1 / m_1 + m_2)\,\mathbf{u_2} \quad \text{(vi)}$$

$$\mathbf{v_2} = (m_2 - m_1 / m_1 + m_2)\,\mathbf{u_2} + (2\, m_1 / m_1 + m_2)\,\mathbf{u_1} \quad \text{(vii)}$$

Case I – If masses are same ($m_1 = m_2$).Then from eq. (i),

$$\mathbf{u_1} + \mathbf{u_2} = \mathbf{v_1} + \mathbf{v_2} \quad \text{(viii)}$$

Adding and subtracting (iii) and (viii), we get

$$\mathbf{v_2} = \mathbf{u_1} \quad \text{and} \quad \mathbf{v_1} = \mathbf{u_2}$$

In head on elastic collision of two bodies the exchange of momentum is maximum when the masses of the bodies are equal ($m_1 = m_2$).

Case II – One of the colliding body is initially at rest like m_2 so $\mathbf{u_2} = 0$.

Hence $\mathbf{v_1} = (m_1 - m_2 / m_1 + m_2)\,\mathbf{u_1}$

and $\mathbf{v_2} = \{2\, m_1 / (m_1 + m_2)\}\,\mathbf{u_1}$ (ix)

Case III - Particle at rest is very massive like m_2. Then $\mathbf{v_1} = -\mathbf{u_1}$.

Case IV- Particle at rest is very light like m_2. Then $\mathbf{v_2} = 2\,\mathbf{u_1}$.

INELASTIC COLLISION IN ONE DIMENSION

Let us consider two particles A and B of masses m_1 & m_2 moving with velocities u_1 and u_2 before collision. After collision the particles strike together and move with the same velocity v.

By conservation of momentum, we get, $m_1\mathbf{u_1} + m_2\mathbf{u_2} = (m_1 + m_2)\mathbf{v}$

$$\mathbf{v} = (m_1\mathbf{u_1} + m_2\mathbf{u_2}) / (m_1 + m_2) \quad \text{(x)}$$

K.E. before collision is $K_1 = ½\, m_1\mathbf{u_1}^2 + ½\, m_2\mathbf{u_2}^2$

And K.E. after collision is $K_2 = ½\,(m_1 + m_2)\mathbf{v}^2$

If m_2 be initially at rest ($u_2 = 0$).Then $K_2 / K_1 = \frac{1}{2}(m_1 + m_2)\mathbf{v}^2 / \frac{1}{2} m_1 \mathbf{u_1}^2$

From eq. (viii), we get, $K_2 / K_1 = \frac{1}{2}(m_1 + m_2)\,\mathbf{\{(}m_1\mathbf{u_1} + m_2\mathbf{u_2}\mathbf{)} / (m_1 + m_2)\mathbf{\}}^2 / \frac{1}{2} m_1 \mathbf{u_1}^2$

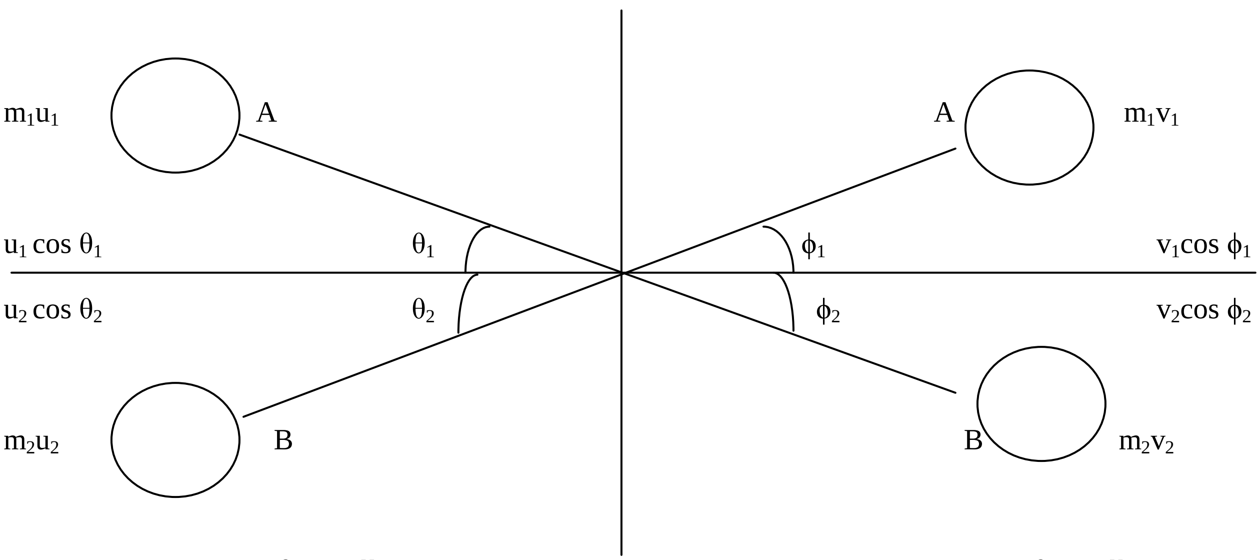
$$K_2 / K_1 = m_1 / (m_1 + m_2) \qquad \text{(xi)}$$

We see that $K_2 < K_1$, therefore the inelastic collision results in a loss of K.E. .

In practice, there is no collision which is perfectly elastic or perfectly inelastic.

ELASTIC COLLISION IN TWO DIMENSION

Let two balls of masses m_1 & m_2 undergoing elastic collision. After collision let A moves with velocity v_1 and makes an angle Φ_1 with x-axis. Similarly let after collision B moves with velocity v_2 and makes an angle Φ_2 with x-axis.



Before collision *After collision*

Before collision-

Momentum of ball A along x-axis = $m_1\mathbf{u_1}\cos\theta_1$

B = $m_2\mathbf{u_2}\cos\theta_2$

Total momentum**,** $\mathbf{P}_{\text{before}} = m_1\mathbf{u_1}\cos\theta_1 + m_2\mathbf{u_2}\cos\theta_2$

After collision-

Momentum of ball A along x-axis = $m_1\mathbf{v_1}\cos\Phi_1$

B = $m_2\mathbf{v_2}\cos\Phi_2$

Total momentum, $\mathbf{P}_{\text{after}} = m_1\mathbf{v_1}\cos\Phi_1 + m_2\mathbf{v_2}\cos\Phi_2$

According to conservation of linear momentum

$$\mathbf{P}_{\text{before}} = \mathbf{P}_{\text{after}}$$

$$m_1\mathbf{u_1}\cos\theta_1 + m_2\mathbf{u_2}\cos\theta_2 = m_1\mathbf{v_1}\cos\Phi_1 + m_2\mathbf{v_2}\cos\Phi_2$$

IMPACT

When two bodies collide with or impinge on each other, this phenomenon of colliding is called impact.

In impact, the velocities of the colliding bodies change in direction as well as magnitude.

COEFFICIENT OF RESTITUTION OF RESIENCE

It is defined as the ratio of relative velocity of separation to relative velocity of approach.

It is denoted by 'e', $e = -(\mathbf{v_2} - \mathbf{v_1}) / (\mathbf{u_2} - \mathbf{u_1})$

It lies between 0 and 1. $0 < e < 1$

For perfectly elastic bodies $e = 1$

Perfectly plastic bodies $e = 0$ (Inelastic bodies)

$e = -\mathbf{v/u}$

Difference b/w Elastic and Inelastic collision

Elastic collision		**Inelastic collision**	
i)	K.E. before = K.E. after	i)	K.E. before ≠ K.E. after
ii)	P.E. is conserved	ii)	P.E. is not conserved
iii)	M.E. is also conserved	iii)	M.E. is not conserved
iv)	K.E. + P.E. = constant	iv)	K.E. + P.E. ≠constant
v)	For perfectly elastic collision e = 1	v)	For perfectly elastic collision e = 0

LOSS OF K.E DURING IMPACT

There is always a loss of K.E. on impact b/w two bodies.

In case direct impact

$$m_1\mathbf{v_1} + m_2\mathbf{v_2} = m_1\mathbf{u_1} + m_2\mathbf{u_2}$$

and $$(\mathbf{v_1} - \mathbf{v_2}) = e(\mathbf{u_1} - \mathbf{u_2})$$

Therefore squaring both sides of both eq. above and adding multiplication of m_1m_2 both sides second equation,

$$(m_1\mathbf{v_1} + m_2\mathbf{v_2})^2 + m_1 m_2(\mathbf{v_1} - \mathbf{v_2})^2 = (m_1\mathbf{u_1} + m_2\mathbf{u_2})^2 + m_1 m_2 e^2(\mathbf{u_1} - \mathbf{u_2})^2$$

$$= (m_1\mathbf{u_1} + m_2\mathbf{u_2})^2 + m_1 m_2(\mathbf{u_1} - \mathbf{u_2})^2 - m_1 m_2(\mathbf{u_1} - \mathbf{u_2})^2 + m_1 m_2 e^2(\mathbf{u_1} - \mathbf{u_2})^2$$

$$= (m_1\mathbf{u_1} + m_2\mathbf{u_2})^2 + m_1 m_2 (\mathbf{u_1} - \mathbf{u_2})^2 - m_1 m_2 (1-e^2) (\mathbf{u_1} - \mathbf{u_2})^2$$

$$(m_1 + m_2)(m_1\mathbf{v_1}^2 + m_2\mathbf{v_2}^2) = (m_1 + m_2)(m_1\mathbf{u_1}^2 + m_2\mathbf{u_2}^2) - m_1 m_2 (1-e^2)(\mathbf{u_1} - \mathbf{u_2})^2$$

$$(m_1\mathbf{v_1}^2 + m_2\mathbf{v_2}^2) = (m_1\mathbf{u_1}^2 + m_2\mathbf{u_2}^2) - m_1 m_2 / (m_1 + m_2)(1-e^2)(\mathbf{u_1} - \mathbf{u_2})^2$$

Therefore loss in K.E.

$$(1/2 m_1\mathbf{u_1}^2 + 1/2 m_2\mathbf{u_2}^2) - (1/2 m_1\mathbf{v_1}^2 + 1/2 m_2\mathbf{v_2}^2) = \tfrac{1}{2} m_1 m_2 /(m_1 + m_2)(1-e^2)(\mathbf{u_1} - \mathbf{u_2})^2$$

This is always + ve. Hence a loss of energy occurs on Impact.

In case of elastic e = 1

Plastic or inelastic e = 0

When $u_1 = u_2$ i.e. bodies travelling same velocities again there is no loss of energy.

CONSERVATIVE FORCE

A force is said to be conservative if the work done by it in moving a particle from one point to another point depends only on these points and not on path followed.

e.g., Electrostatic force, Gravitational force, Central force, restoring force and Lorentz force.

Consider a particle of mass 'm' moving from point A to B. The particle can travel several path I, II, III. In going from A to B, if the work done against force on each path is same. Then the force acting on the particle is conservative.

Let dw be the work done for small displacement d**r** is given by

$$dw = \mathbf{F}.d\mathbf{r}$$

Hence the total work done for displacement from A to B

$$W = \int_A^B dw = \int_A^B F.d\boldsymbol{r}$$

For Conservative force, $\int_A^B F.d\boldsymbol{r} = \int_A^B F.d\boldsymbol{r} = \int_A^B F.d\boldsymbol{r}$

Path (I) Path (II) Path (III)

$$\int_I dw = \int_{II} dw = \int_{III} dw$$

This region in which a particle experiences a conservative force is called as conservative force field.

Properties:

1. Work done by a conservative force around a closed path is always zero.
   Mathematically $\oint \boldsymbol{F}.d\boldsymbol{r} = \oint dw = o$

Proof: Let W_{AB} be the work done in taking a body from A to B

$$W_{AB} = \int_A^B F.d\boldsymbol{r} \qquad (1)$$

Similarly work done in taking the body from B to A is given by

$$W_{BA} = \int_B^A F.d\boldsymbol{r} \qquad (2)$$

The net work done in taking the body from A to B and then B to A (closed path) will be

$$W_{AB} + W_{BA} = \oint \boldsymbol{F}.d\boldsymbol{r} = \int_A^B F.d\boldsymbol{r} + \int_B^A F.d\boldsymbol{r}$$

$$= \int_A^B F.d\boldsymbol{r} - \int_A^B F.d\boldsymbol{r}$$

$$\oint \boldsymbol{F}.d\boldsymbol{r} = 0$$